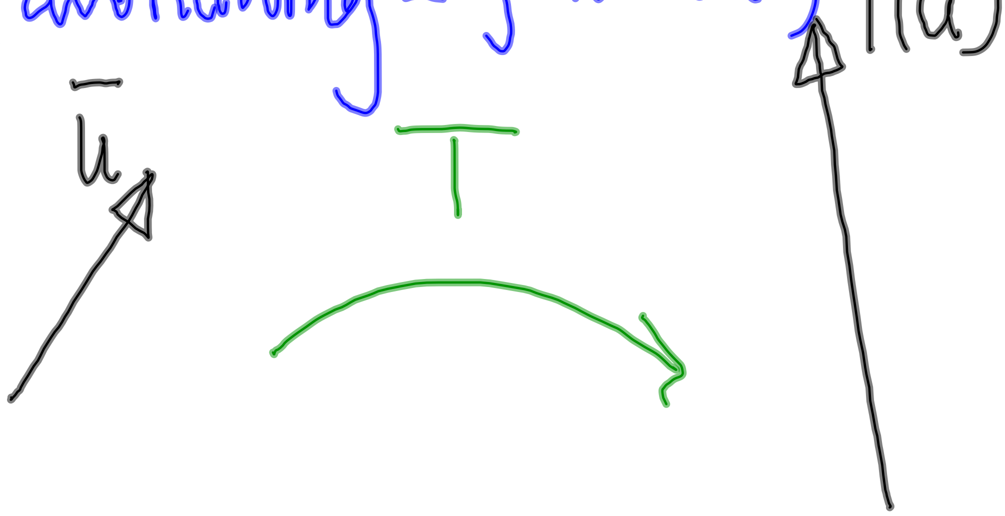
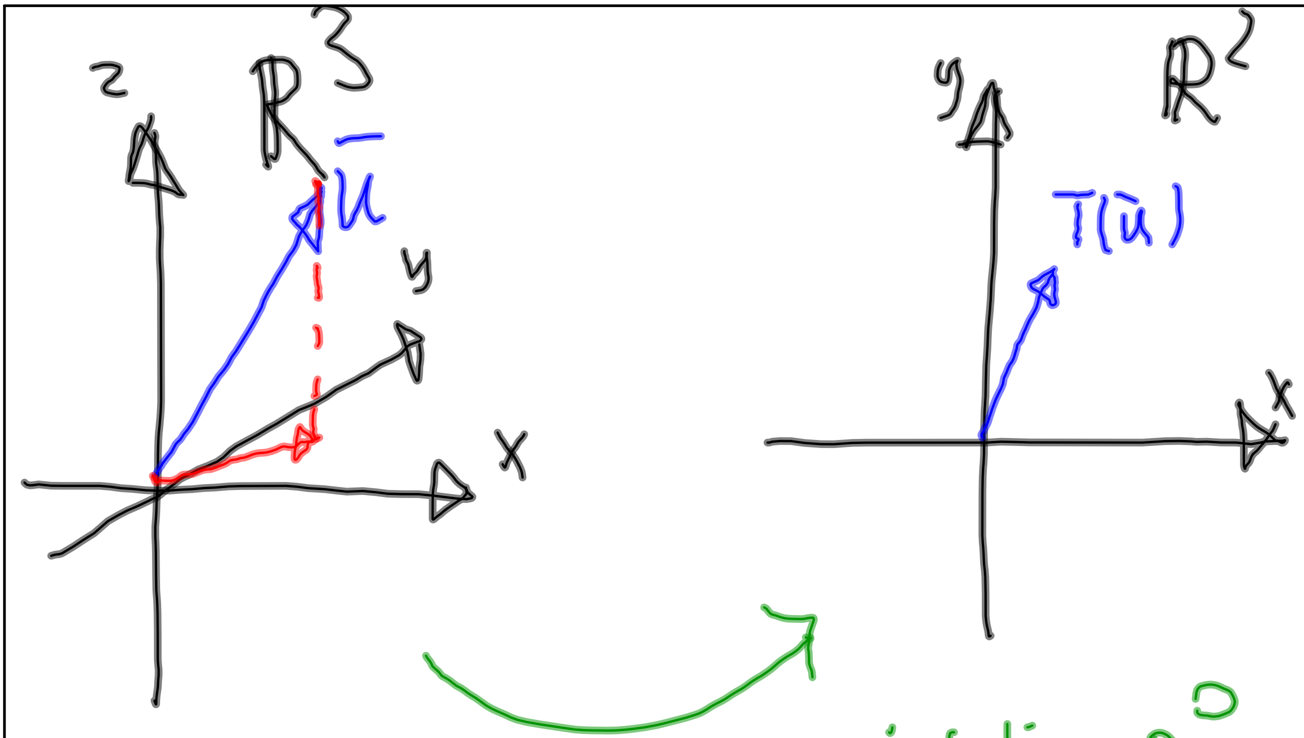
Linjära avbildningar

(avbildning = funktion)



nov 10-07:45



T = projektion på xy-planet

Projektion bevarar summan och mult. med skalär

nov 10-07:45

Väsentligt att den
bevarar (eller respekterar)

- addition $\bar{u}+\bar{v}$
- mult. m. skalär $a\bar{u}$

nov 10-07:45

$$\mathrm{Proj}_{\bar{v}}(\bar{u}+\bar{w}) = \mathrm{Proj}_{\bar{v}}(\bar{u}) + \mathrm{Proj}_{\bar{v}}(\bar{w})$$

$$\mathrm{Proj}_{\bar{v}}(a\bar{u}) = a\,\mathrm{Proj}_{\bar{v}}(\bar{u})$$

nov 10-07:45

Skalärprodukt

$\bar{u} \cdot \bar{v}$

$(\bar{u}+\bar{w}) \cdot \bar{v} = \bar{u} \cdot \bar{v} + \bar{w} \cdot \bar{v}$

$(a\bar{u}) \cdot \bar{v} = a(\bar{u} \cdot \bar{v})$

nov 10-07:45

Vektorprodukt

$$(\bar{u}+\bar{w})\times\bar{v}=\bar{u}\times\bar{v}+\bar{w}\times\bar{v}$$

$$(a\bar{u})\times\bar{v}=a(\bar{u}\times\bar{v})$$

nov 10-07:45

Exempel på icke-linjära funktioner (avbildningar)

- kubiskt $f(x)=x^3$

$$f(x+y)=(x+y)^3=x^3+3x^2y+3xy^2+y^3$$

$$\neq f(x)+f(y)=x^3+y^3$$

nov 10-07:45

forts.

- exponentialfunktionen

$$e^{x+y} = e^x \cdot e^y \neq e^x + e^y$$

nov 10-08:15

forts

- trigonometriska funktioner

$$\sin(x+y) = \sin x \cos y + \sin y \cos x$$
$$\neq \sin x + \sin y$$
$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$
$$\neq \cos(x) + \cos(y)$$

nov 10-08:16

fo<u>rts.</u>

- konstanter $f(x) = a$

$$f(x+y) = a \neq a + a = f(x) + f(y)$$

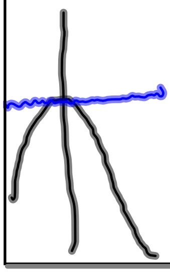
(bara linjär om $a = 0$)

nov 10-08:18

Senare kommer vi approximera icke-linjära funktioner med linjära

$0+1\cdot x$

$\sin x \approx x$ för små x



$\cos x \approx 1$ för små x

$\cos x \approx 1 + 0\cdot x$

Vi kan alltid skriva
linjära avbildningar med
matriser

$$T(\bar{u}) = A\bar{u}$$

($\bar{u}$ tänkt som en kolonn)

nov 10-08:25

Exempel 1) $f(x,y)=3x-2y$

$$\begin{pmatrix} 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=3x-2y$$

t

2) $g(x,y)=(3x-2y,\ x+y)$

$$\begin{pmatrix} 3x-2y \\ x+y \end{pmatrix}=\begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

nov 10-08:26

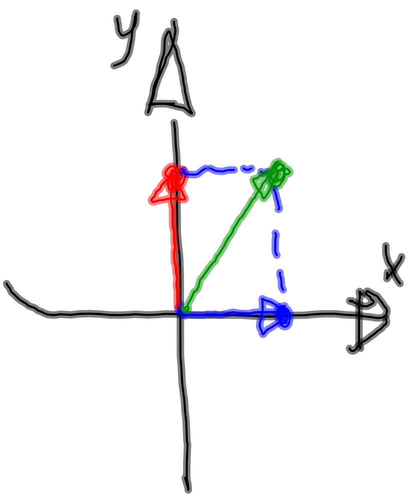
<u>Exempel</u> Projektion på xy-planet

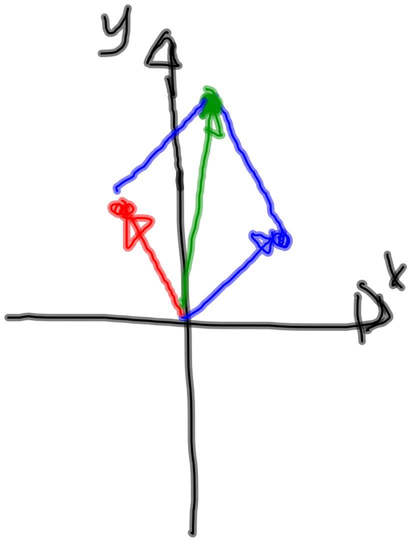
$$\left.\begin{array}{l} T\begin{pmatrix}1\\0\\0\end{pmatrix}=\begin{pmatrix}1\\0\\ \end{pmatrix} \\ T\begin{pmatrix}0\\1\\0\end{pmatrix}=\begin{pmatrix}0\\1\end{pmatrix} \\ T\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}\end{array}\right\} \Rightarrow A=\begin{pmatrix}1&0&0\\0&1&0\end{pmatrix}$$

$$A\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1&0&0\\0&1&0\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}x\\y\end{pmatrix}$$

nov 10-08:34

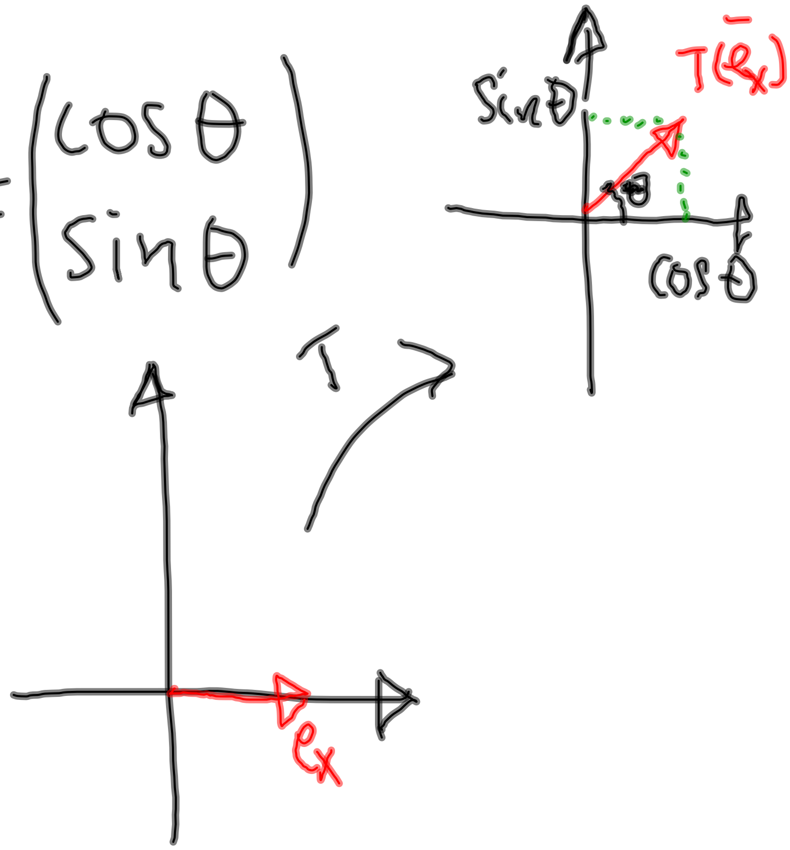
Rotation av planet





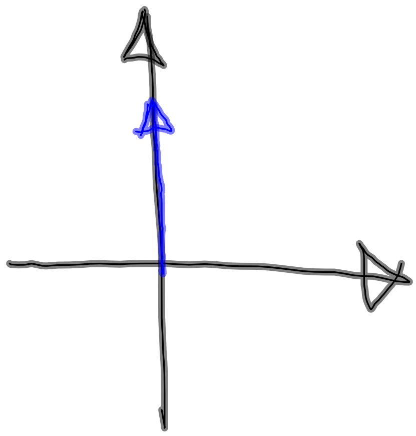
nov 10-08:38

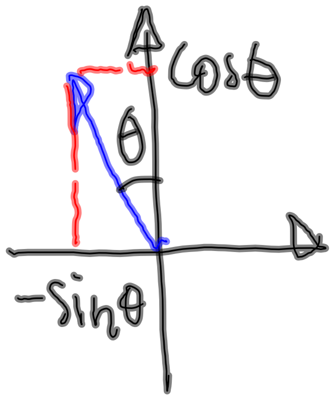
$$T(\bar{e}_x) = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$



nov 10-08:42

$$T(\bar{e}_y) = \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$$





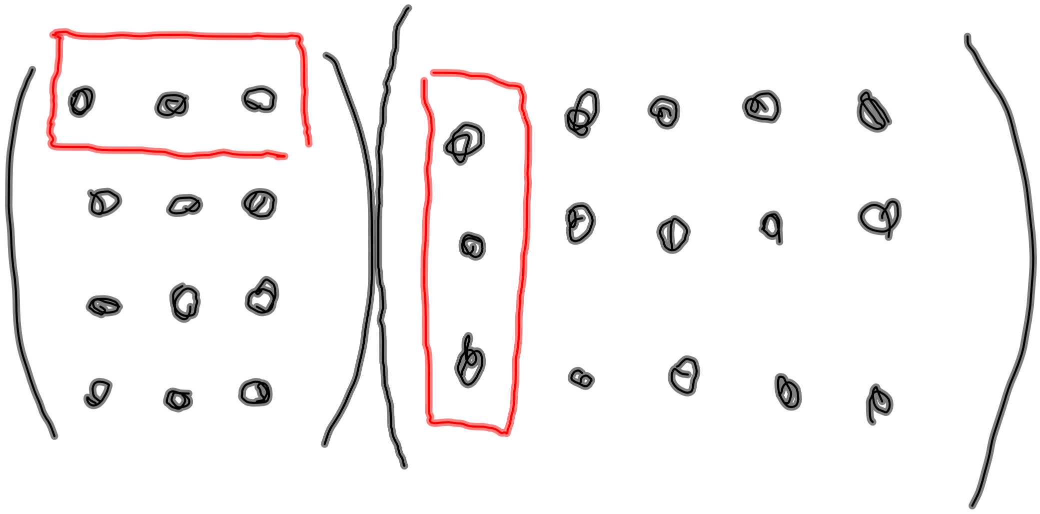
nov 10-08:44

Ex: $\theta = 90^\circ$

$$\begin{pmatrix} \cos 90^\circ & -\sin 90^\circ \\ \sin 90^\circ & \cos 90^\circ \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

nov 10-08:46

Matrismult.



nov 10-09:04

Tolka matris-multiplikation som

sammansättning

$$f(g(x)) = fg(x)$$

$$= f \circ g(x)$$

nov 10-09:07

$$T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$$

$$S : \mathbb{R}^m \longrightarrow \mathbb{R}^k$$

$$\mathbb{R}^n \xrightarrow{T} \mathbb{R}^m \xrightarrow{S} \mathbb{R}^k$$

nov 10-09:09

$$\begin{pmatrix} \boxed{\cos\varphi & -\sin\varphi} \\ \sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$\cos\varphi \cdot \cos\theta + (-\sin\varphi)\sin\theta$$

$$= \cos\varphi\cos\theta - \sin\varphi\sin\theta$$

$$= [\text{additionsformel.}]$$

$$= \cos(\varphi + \theta)$$

nov 10-09:15

Projektion

$$\mathrm{Proj}_{\bar{v}}\bar{u} = \frac{\bar{u}\circ\bar{v}}{|\bar{v}|^2}\bar{v}$$

Välj en vektor $\bar{v}$!

$$\bar{v} = \begin{pmatrix} 5 \\ 8 \\ -13 \end{pmatrix}, \quad |\bar{v}|^2 = 258$$

nov 10-09:19

Bestäm matrisen genom att beräkna avbildningen för $\bar{e}_x$, $\bar{e}_y$ och $\bar{e}_z$.

$$\mathrm{Proj}_{\bar{v}}\,\bar{e}_x = \frac{1}{258}(\bar{e}_x \cdot \bar{v})\bar{v}$$

$$= \frac{1}{258} \cdot 5 \cdot \bar{v} = \frac{5}{258}\begin{pmatrix} 5 \\ 8 \\ -13 \end{pmatrix}$$

nov 10-09:21

$$\mathrm{Proj}_{\bar{v}}\,\bar{e}_y = \frac{8}{258}\begin{pmatrix}5\\8\\-13\end{pmatrix}$$

$$\mathrm{Proj}_{\bar{v}}\,\bar{e}_2 = \frac{-13}{258}\begin{pmatrix}5\\8\\-13\end{pmatrix}$$

Alltså

$$A = \frac{1}{258}\begin{pmatrix}25 & 40 & -65\\ 40 & 64 & -104\\ -65 & -104 & 169\end{pmatrix}$$

nov 10-09:24

A är <u>symmetrisk</u>

dvs $A = A^t$

$$\mathrm{Proj}_{\bar{v}}\bar{u} = \frac{1}{|\bar{v}|^2}(\bar{u}\cdot\bar{v})\bar{v}$$

$$= \frac{1}{|\bar{v}|^2}\bar{v}(\bar{v}\cdot\bar{u})$$

$$= \frac{1}{|\bar{v}|^2}\bar{v}\,\bar{v}^t\bar{u}$$

nov 10-09:28

Alltså är matrisen för

$\mathrm{Proj}_{\bar{v}}$

$$\frac{1}{|\bar{v}|^2}\bar{v}\,\bar{v}^t$$

$$= \frac{1}{258}\begin{pmatrix} 5 \\ 8 \\ -13 \end{pmatrix}\begin{pmatrix} 5 & 8 & -13 \end{pmatrix}$$

nov 10-09:32

$\bar{v}\bar{v}^t$ är

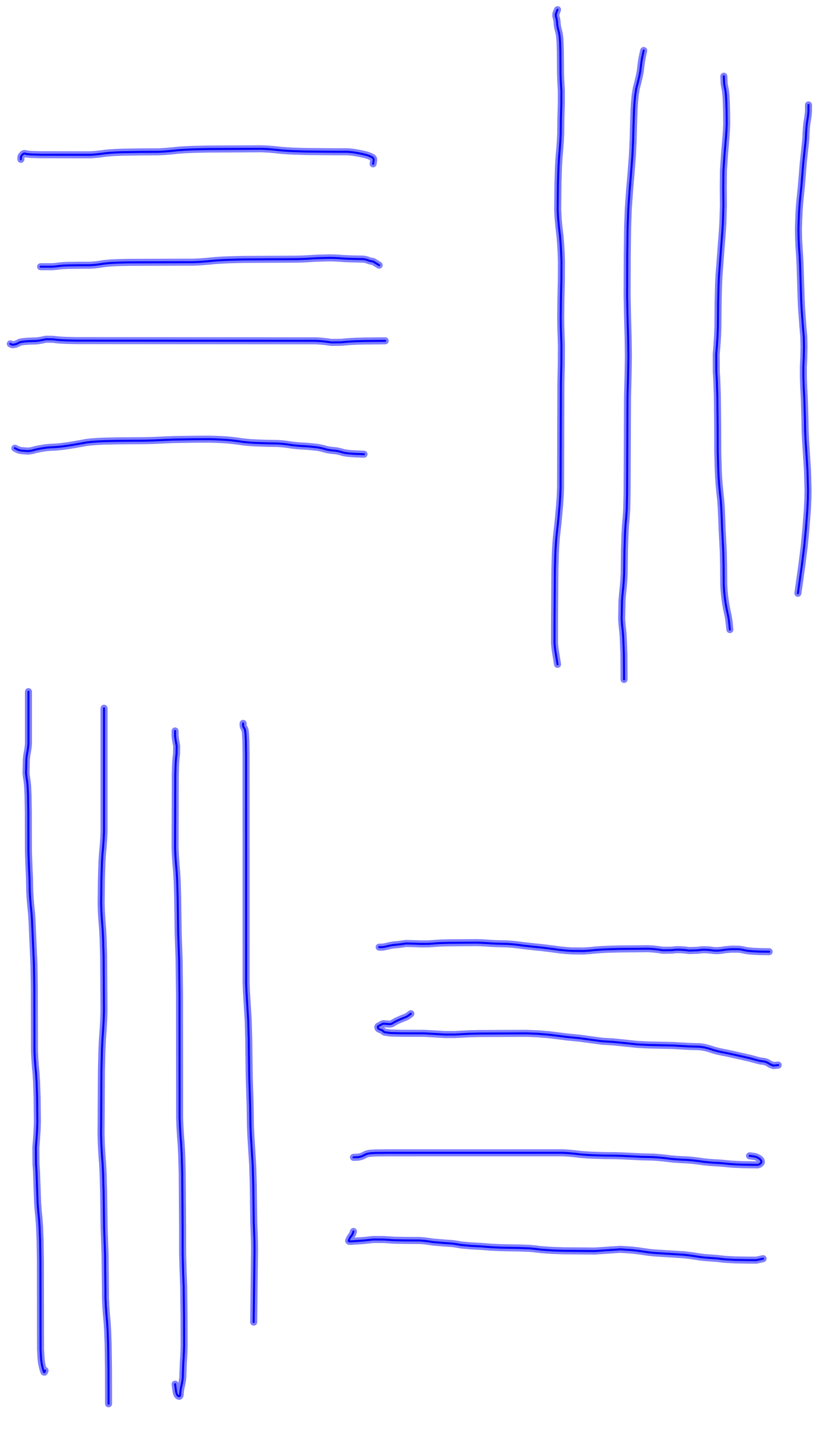
alltid symmetrisk

$(\bar{v}\bar{v}^t)^t =$

$= (\bar{v}^t)^t \bar{v}^t = \bar{v}\bar{v}^t$

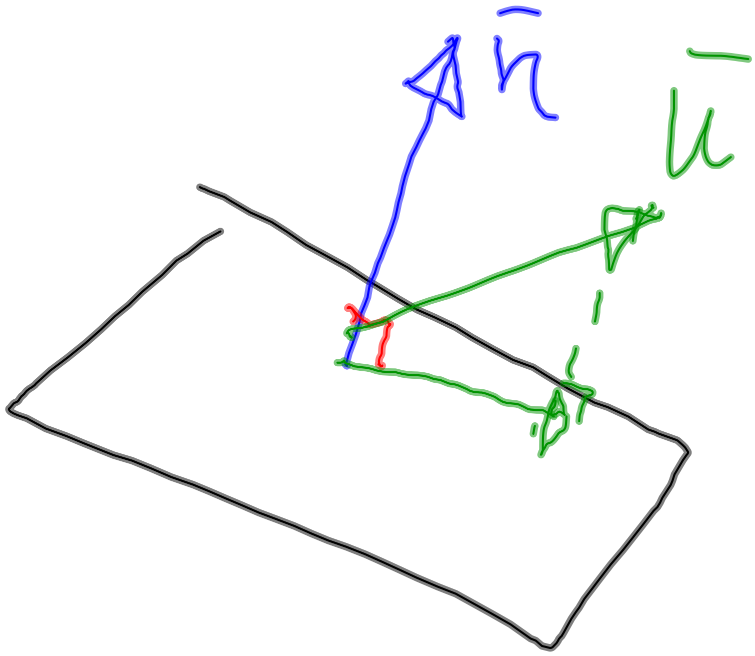
$(AB)^t = B^t A^t$

nov 10-09:33

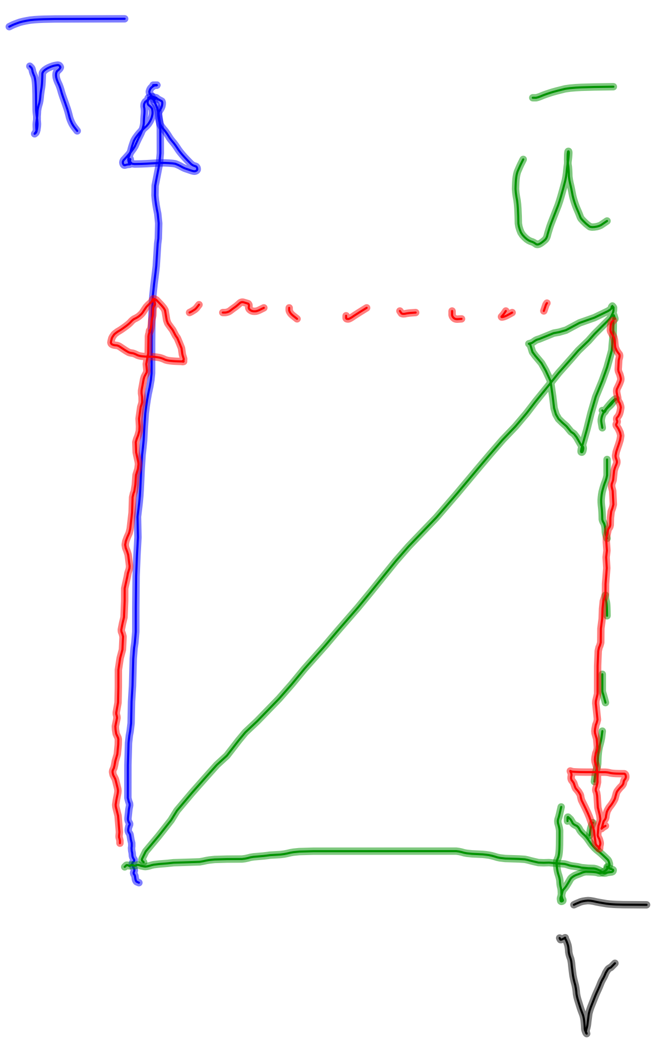


nov 10-09:36

Projektion på plan i rymden



nov 10-09:37



$$\bar{v} = \bar{u} - \mathrm{Proj}_{\bar{n}}\,\bar{u}$$

nov 10-09:38

Om T är proj. på planet med normal $\bar{n}$. (genom origo) så ges T av

$$T(\bar{u}) = \bar{u} - \mathrm{Proj}_{\bar{n}}\bar{u}$$

nov 10-09:41

Matrisen blir

$$I_3 - \frac{1}{|\bar{n}|^2}\bar{n}\bar{n}^t$$

Ex: $\bar{n} = (1, 0, -1)$

$\Rightarrow |\bar{n}|^2 = 1^2 + 0^2 + (-1)^2 = 2$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}\begin{pmatrix} 1 & 0 & -1 \end{pmatrix}$$

nov 10-09:42

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$$

nov 10-09:44